

## SUPPLEMENTAL MATERIALS

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# Laboratory Measurements of Hydraulic Jacking Uplift Pressure at Offset Joints and Cracks

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## Additional Uplift Due to Detached Jets

Eqs. (5)-(9) predict the uplift pressure head for jets that remain attached to the chute floor downstream from an offset into the flow. If the flow detaches from the floor due to a large offset that forces the flow into the air so that the jet breaks up before landing downstream, the uplift pressure will be increased due to the momentum thrust needed to deflect the jet. Some of the increased pressure on the chute floor is transmitted into the joint.

Data collected from detached jets were used to determine the uplift pressure increase in relation to  $h_v$ ,  $h_v^*$ , and the normalized uplift pressure head for the attached condition. The best relation for predicting the additional uplift is shown in Fig. S1:

$$\frac{\Delta H_{\text{det}}}{h_v^*} = 0.0382 \left( \frac{y}{s} \frac{\beta^{1.35}}{F^{0.5}} \right)^{0.90} \quad (\text{S1})$$

which is fit using data collected in the range  $0 < \beta < 5.5$  with  $F$  being the Froude number immediately upstream from the joint. Larger values of  $\beta$  produce a smaller increase. The maximum value of  $\beta$  for which a detached jet could be tested was 9.71. Once the increase due to detachment is computed, the total uplift would be  $\Delta H + \Delta H_{\text{det}}$ , with  $\Delta H$  obtained from Eq. (5).

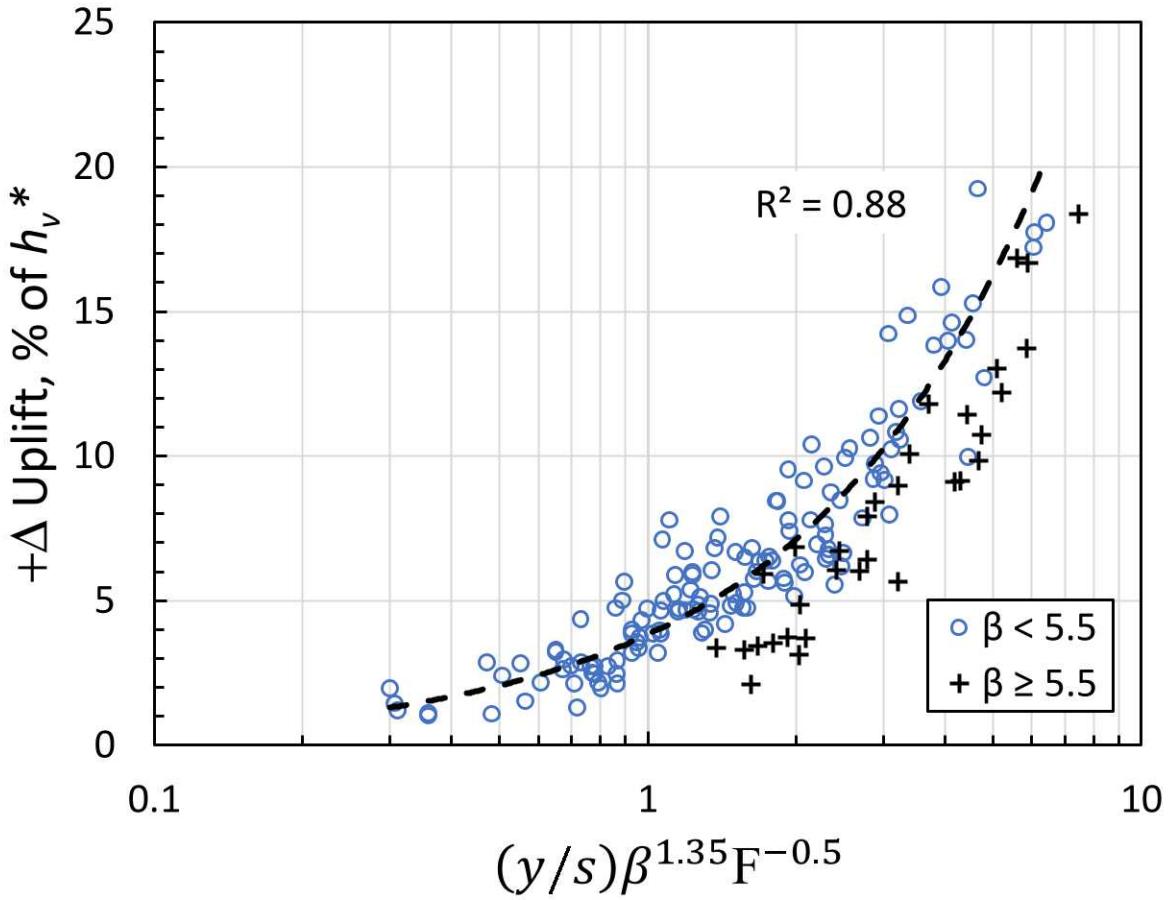
An alternative relation for predicting the additional uplift is

$$\frac{\Delta H_{\text{det}}}{h_v} = 0.0240 \left( \frac{y}{s} \frac{\beta^{1.35}}{F^{0.5}} \right)^{0.724} \quad (\text{S2})$$

illustrated in Fig. S2. The data in Figs. S1-S2 were all obtained at detached flow conditions for which a corresponding attached flow case could be established and tested. Either equation can also be applied to very shallow flow depths for which attached flow is impossible to maintain. Eight tests were conducted in such conditions, with offsets  $h = 8$  to 10 mm and four values of  $\beta$ : 0.07, 0.47, 0.97 and 9.0. Observed uplift pressures were within 10% of predicted values from Eq. (S1) for most of the tests, except the  $\beta = 9.0$  case and two low-flow tests with  $y/h < 1$ . Errors in these cases were up to  $\pm 40\%$ , but these are all situations that produce very little actual uplift. For the cases with  $y/h < 1$ , the predicted uplift was computed assuming  $\alpha^* = 1.0$ , since Eq. (4) computes  $\alpha^*$  values greater than 1 that are physically inconsistent with a flow that is thinner than the offset height.

One can estimate whether the chute flow will be naturally detached by applying techniques for estimating the launch angle and throw distance of jets at spillway aerator ramps (Falvey 1990; Pfister and Hager 2010). The chute slope is designated  $\theta (+15^\circ$  for the laboratory chute), and the perpendicularly offset face is positioned at  $\theta_o = \theta - 90^\circ = -75^\circ$ , with the negative angle indicating that the offset would direct the jet upward, above horizontal (Fig. S3). The launch angle of the jet is  $\theta_e = \theta_i(1 - A_r) + \theta_o A_r$ , with the weighting factor  $A_r = 0.48(y/h)^{-0.4}$  for a perpendicular offset (equation fit to curve in Falvey 1990, originally from Pan et al. 1980). Note that the sign of  $\theta_e$  should be preserved; it will be negative if the launch angle is above horizontal. The launch angle relative to the chute floor is then  $\alpha' = \theta - \theta_e$ . The Froude number at the launch point is  $F = V/(gy)^{0.5}$ . The throw distance of the jet parallel to the chute floor can be estimated from an empirical equation developed by Pfister and Hager (2010), which is simplified for the case of no offset downstream from the launch point to  $L = h(0.77)F^2(\tan \alpha')(1+\sin \theta)^{1.5}$ . In the air, the thinning of the intact core of the jet over this throw distance can be estimated as  $2T_iL$

where  $T_i$  is a measure of vertical turbulence intensity (perpendicular to chute floor) relative to mean velocity that should range from about 0.03 to 0.06 (Falvey 1990; Auel et al. 2014). The thickness of the remaining jet core at the landing point is then  $y - 2T_i L$ . If the computed core thickness is near zero or negative, the jet can be considered disintegrated at the landing point, which will allow air penetration and detached flow. This calculation was applied to the test data and was found to provide a reasonable indication of attached vs. detached jet conditions; however, regardless of the value assumed for  $T_i$ , there were a few poorly predicted cases associated with large gap widths, small offset heights, or both (i.e., large  $\beta$ ). For these situations, the launch angle may be poorly predicted because the method assumes a solid bottom boundary upstream from the offset. This is consistent with the difference in additional detached uplift seen in Fig. S1 and S2 for  $\beta$  values above 5.5, and the range of  $\beta$  values associated with the offset-height scale effect discussed earlier.

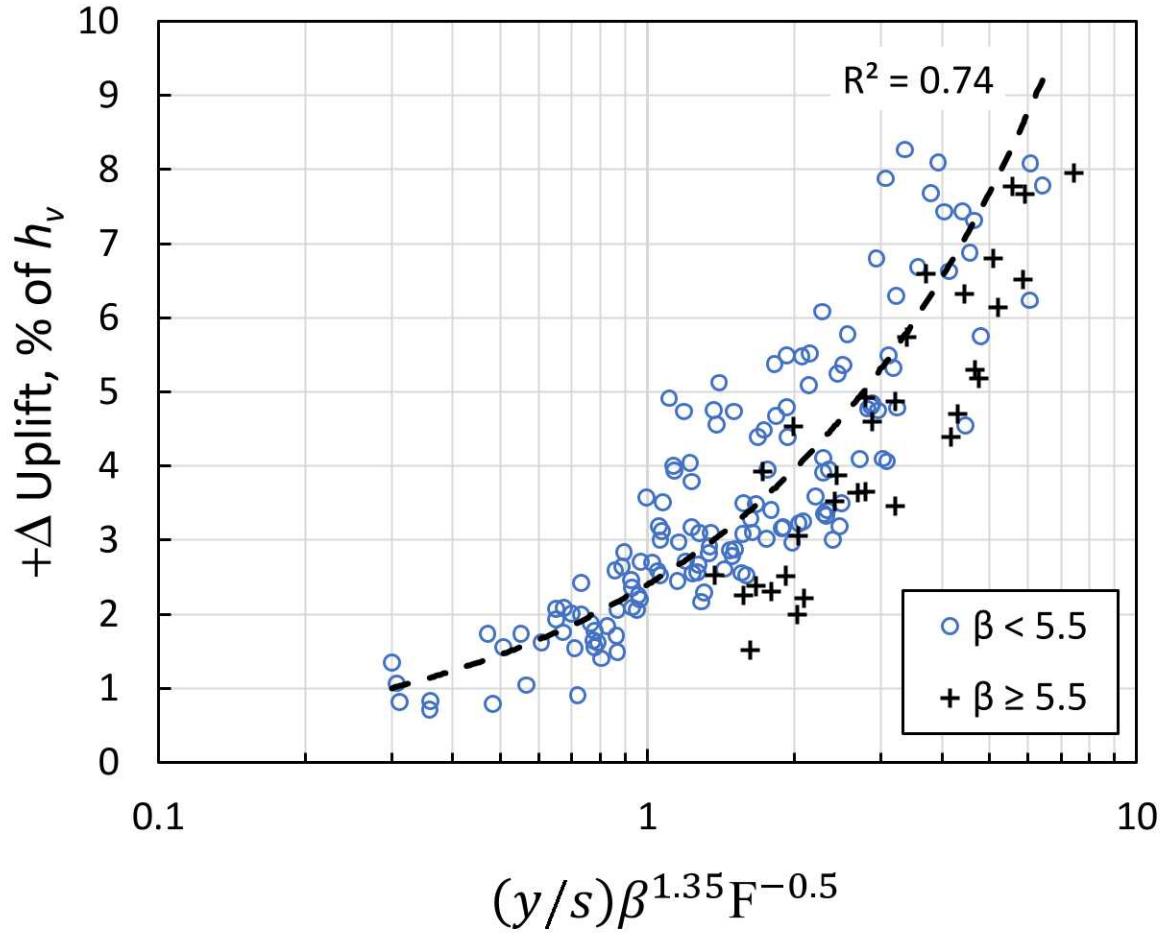


**Fig. S1.** Additional uplift due to jet detachment, relative to  $h_v^*$ . This relation was developed using data in the range of  $0.29 < y/s < 178$ ,  $0.044 < \beta < 9.71$ , and  $7.6 < F < 13.1$ .

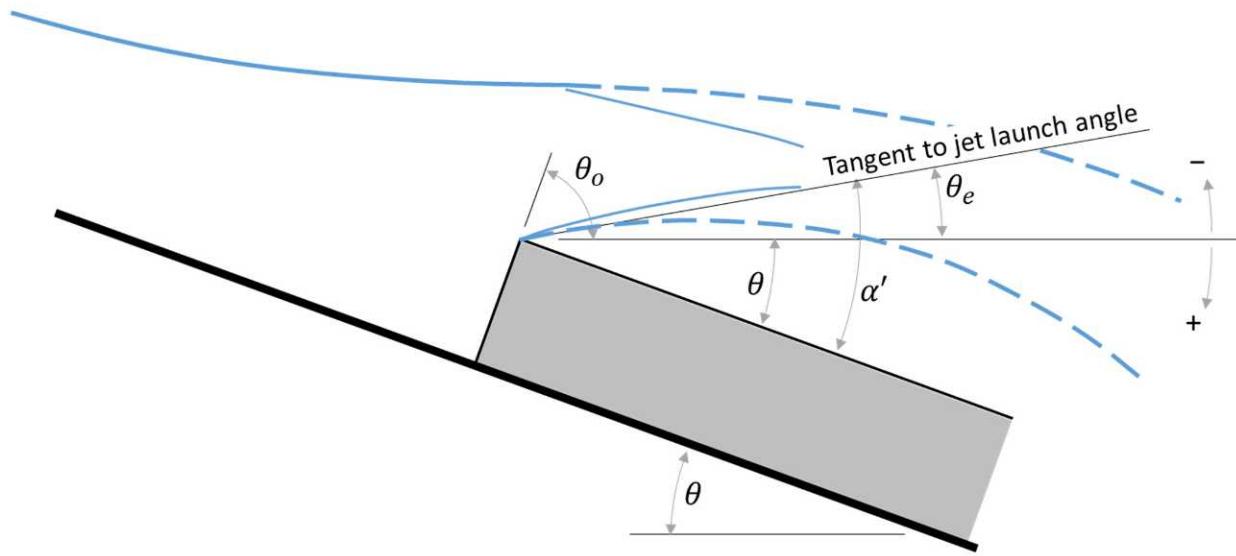
Using the estimated launch angle, another empirical relation for the additional uplift is

$$\frac{\Delta H_{\text{det}}}{\Delta H} = 0.000140 \left( \frac{(\tan \alpha')^{0.75}}{e^{0.1\beta}} \right)^{-2.52} \quad (\text{S3})$$

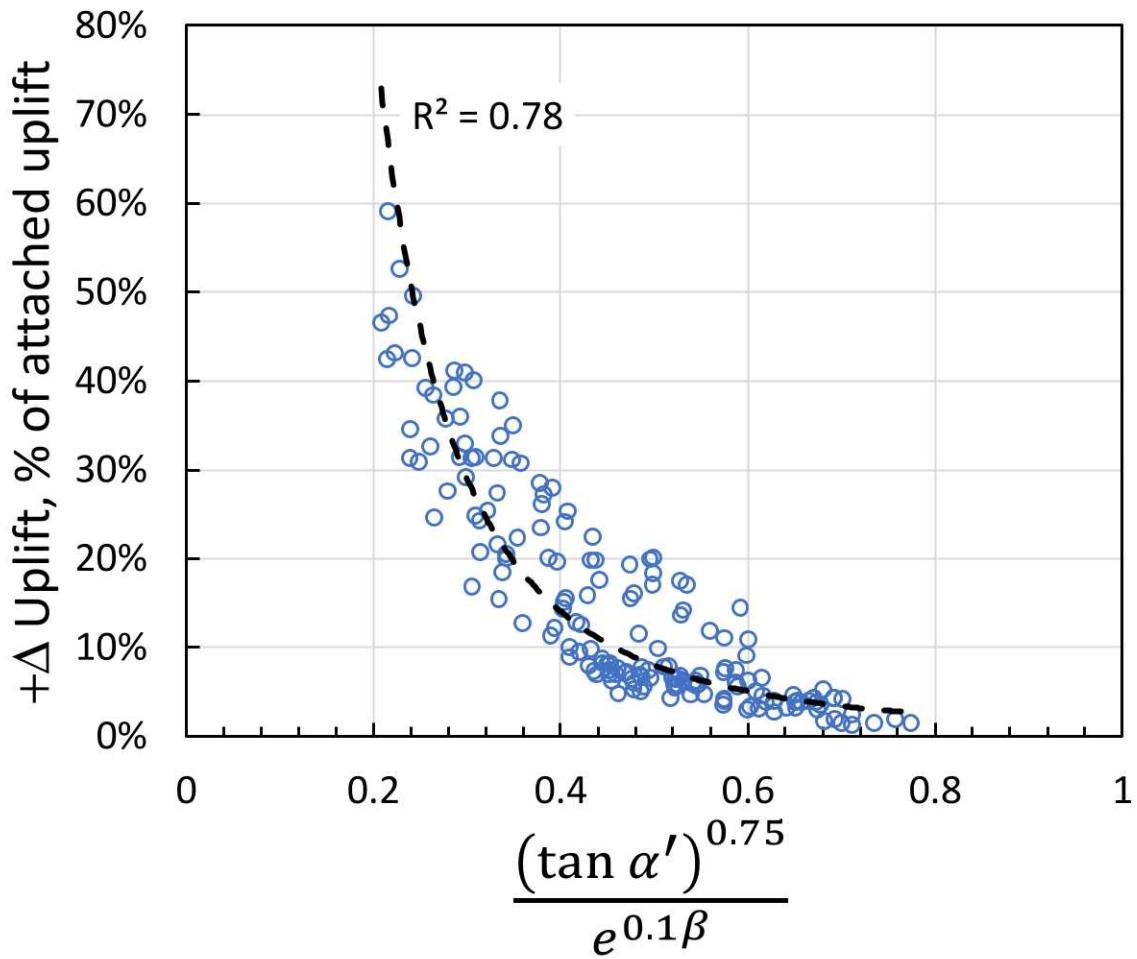
where  $\Delta H$  is the uplift calculated for attached flow conditions. This relation is shown in Fig. S4. Note that no difference was noted in this relation for different ranges of  $\beta$ .



**Fig. S2.** Additional uplift due to jet detachment, relative to  $h_v$ . This relation was developed using data in the range of  $0.29 < y/s < 178$ ,  $0.044 < \beta < 9.71$ , and  $7.6 < F < 13.1$ .



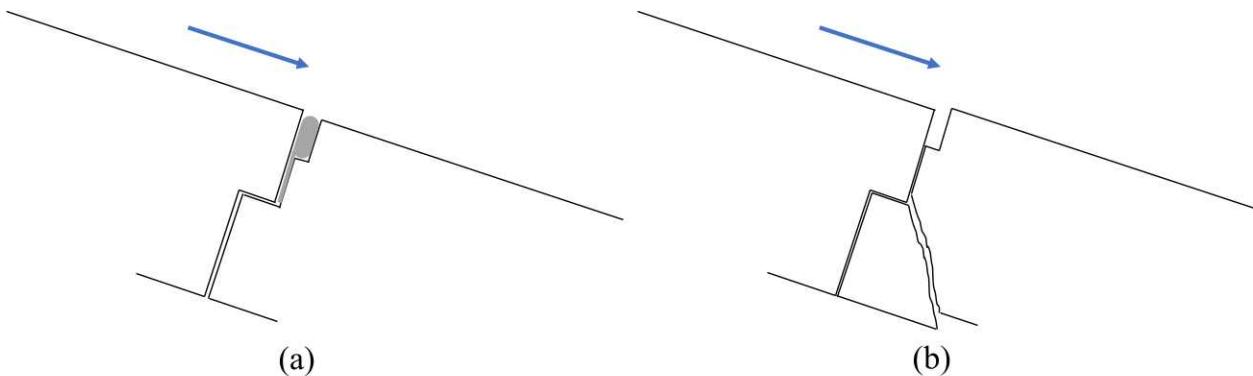
**Fig. S3.** Schematic diagram and definition of variables for jet launching free from an offset into the flow.



**Fig. S4.** Additional uplift due to jet detachment, relative to attached uplift, with  $e = 2.718$ , the base of natural logarithms. This relation is applicable for  $14^\circ < \alpha' < 39^\circ$  and  $0.044 < \beta < 9.71$ .

## Expanded-Gap Joint Concept

Because maximum uplift pressure occurs as  $s$  and  $\beta$  approach zero, a joint designed to prevent this is desirable. One concept is illustrated in Fig. S5, incorporating an initial offset away from the flow, a keyway to prevent differential movement of the downstream slab into the flow, and a joint filler to seal the joint. Other features, such as dowel pins and embedded waterstops could also be included but are not essential to the concept. With this joint design, even if the keyway fails so that the downstream slab can become offset into the flow, the joint filler material is removed, and the joint is compressed to a zero gap at the base, the expanded gap will keep  $\beta$  from being reduced to zero. Testing confirms that the effective  $\beta$  value of a joint is determined by the gap width at the chute surface, so maintaining a gap width greater than zero there will produce less uplift than a comparable offset joint having  $\beta$  near zero. Depending on the anchorage system capacity planned for a chute and the size of potential offsets that might develop, the designer can use the uplift equations to determine the expanded gap size needed to keep uplift within a manageable range.



**Fig. S5.** Concept for a joint with an expanded gap width at the chute surface, showing (a) the constructed joint with an initial offset away from the flow, a keyway to prevent differential movement of the downstream slab into the flow, and joint filler material (light gray); and (b) the joint after failure of the keyway, washout of filler material, compression of the joint, and offsetting of the downstream slab into the flow. In this offset condition, the expanded gap at the chute surface still prevents a  $\beta=0$  condition.

## Experimental Data

**Table S1.** Test data for uplift pressure at regular square-edged joints. Sorted by  $\beta$ ,  $h$ , and  $q$ , with detached-only and rough-floor tests at bottom.

Offset $h$	Gap $s$	$\beta$	Discharge per unit width				Depth $y$	Velocity $V$	$N$	$\alpha$	$\alpha^*$	Channel velocity head $h_v$	Boundary layer velocity head $h_v^*$	Attached uplift $\Delta H$	Total detached uplift $\Delta H + \Delta H_{det}$	Approach flow surface
			mm	mm	-	$m^2/s$	cm	m/s	-	-	-	m	m	m	m	-
10.19	0.45	0.044	0.178	3.09	5.77	8.03	1.035	0.684	1.699	1.162	0.897	0.911	0.911	0.911	acrylic	
10.19	0.45	0.044	0.273	4.42	6.17	8.28	1.033	0.607	1.942	1.179	1.003	1.036	1.036	1.036	acrylic	
10.19	0.45	0.044	0.597	7.98	7.48	8.55	1.032	0.501	2.853	1.429	1.343	1.424	1.424	1.424	acrylic	
10.33	0.47	0.045	0.174	3.02	5.78	7.81	1.037	0.687	1.705	1.171	0.937	0.960	0.960	0.960	acrylic	
10.33	0.47	0.045	0.316	4.98	6.35	8.58	1.031	0.595	2.058	1.224	1.115	-	-	-	acrylic	
10.33	0.47	0.045	0.593	7.80	7.60	9.67	1.025	0.548	2.944	1.612	1.504	1.580	1.580	1.580	acrylic	
10.38	0.80	0.077	0.145	2.62	5.55	8.45	1.032	0.744	1.569	1.167	0.883	0.900	0.900	0.900	acrylic	
10.38	0.80	0.077	0.175	3.01	5.81	8.40	1.033	0.706	1.719	1.213	0.957	0.969	0.969	0.969	acrylic	
10.38	0.80	0.077	0.244	4.04	6.04	8.77	1.030	0.647	1.862	1.205	1.054	1.083	1.083	1.083	acrylic	
10.38	0.80	0.077	0.310	4.54	6.82	8.61	1.031	0.617	2.375	1.465	1.342	1.383	1.383	1.383	acrylic	
10.38	0.80	0.077	0.327	5.06	6.46	8.82	1.030	0.601	2.129	1.280	1.155	1.196	1.196	1.196	acrylic	
10.38	0.80	0.077	0.595	7.66	7.77	9.28	1.027	0.539	3.075	1.656	1.541	-	-	-	acrylic	
10.38	0.80	0.077	0.749	8.89	8.43	9.72	1.025	0.528	3.620	1.912	1.834	1.923	1.923	1.923	acrylic	
9.88	1.49	0.15	0.125	2.33	5.36	8.63	1.031	0.765	1.466	1.122	0.807	0.819	0.819	0.819	acrylic	
9.88	1.49	0.15	0.235	3.91	6.01	8.42	1.032	0.632	1.841	1.164	0.993	1.031	1.031	1.031	acrylic	
9.88	1.49	0.15	0.569	6.01	9.46	8.06	1.035	0.528	4.567	2.413	2.181	2.301	2.301	2.301	acrylic	
9.88	1.49	0.15	0.572	7.65	7.48	8.81	1.030	0.513	2.851	1.463	1.356	1.444	1.444	1.444	acrylic	
10.26	1.59	0.16	0.165	2.90	5.68	8.14	1.034	0.705	1.648	1.162	0.923	-	-	-	acrylic	
10.26	1.59	0.16	0.370	5.53	6.69	8.42	1.032	0.567	2.284	1.294	1.195	-	-	-	acrylic	
10.26	1.59	0.16	0.595	7.76	7.67	9.08	1.028	0.527	2.997	1.579	1.509	-	-	-	acrylic	
9.96	2.49	0.25	0.240	3.97	6.06	8.40	1.033	0.630	1.873	1.180	0.910	0.939	0.939	0.939	acrylic	
9.96	2.49	0.25	0.406	5.09	7.97	8.82	1.030	0.591	3.238	1.915	1.606	1.683	1.683	1.683	acrylic	
9.96	2.49	0.25	0.407	5.99	6.79	8.26	1.034	0.539	2.354	1.268	1.135	1.209	1.209	1.209	acrylic	
9.96	2.49	0.25	0.598	7.84	7.63	8.10	1.035	0.482	2.972	1.432	1.370	1.467	1.467	1.467	acrylic	
10.37	2.79	0.27	0.153	2.67	5.73	8.13	1.035	0.729	1.673	1.220	0.865	0.879	0.879	0.879	acrylic	
10.37	2.79	0.27	0.180	3.07	5.88	7.91	1.036	0.687	1.760	1.210	0.925	0.943	0.943	0.943	acrylic	
10.37	2.79	0.27	0.299	4.80	6.22	8.23	1.034	0.591	1.972	1.166	1.029	1.073	1.073	1.073	acrylic	
10.37	2.79	0.27	0.306	4.80	6.37	8.00	1.036	0.583	2.071	1.207	1.073	1.116	1.116	1.116	acrylic	
10.37	2.79	0.27	0.424	6.27	6.77	8.84	1.030	0.560	2.338	1.308	1.162	1.213	1.213	1.213	acrylic	
10.37	2.79	0.27	0.595	7.71	7.72	9.30	1.027	0.538	3.040	1.636	1.475	1.552	1.552	1.552	acrylic	

Offset	Gap <i>h</i>	$\beta$	Discharge per unit width			Depth <i>y</i>	Velocity <i>V</i>	<i>N</i>	$\alpha$	$\alpha^*$	Channel velocity head <i>h<sub>v</sub></i>	Boundary layer velocity head <i>h<sub>v*</sub></i>	Attached uplift $\Delta H$	Total detached uplift $\Delta H + \Delta H_{det}$	Approach flow surface
			<i>s</i>	<i>m</i>	<i>m</i> <sup>2</sup> /s										
			mm	mm	-										
10.37	2.79	0.27	0.749	8.80	8.51	9.77	1.025	0.532	3.696	1.965	1.764	1.876	acrylic		
3.70	1.58	0.43	0.042	1.13	3.70	7.68	1.038	0.672	0.697	0.468	0.319	0.331	acrylic		
3.70	1.58	0.43	0.054	1.33	4.11	8.28	1.033	0.651	0.861	0.560	0.385	0.401	acrylic		
3.70	1.58	0.43	0.109	2.10	5.20	8.08	1.035	0.543	1.378	0.748	0.602	0.637	acrylic		
3.70	1.58	0.43	0.225	3.73	6.03	8.38	1.033	0.452	1.851	0.837	0.769	0.842	acrylic		
3.70	1.58	0.43	0.393	5.73	6.86	9.10	1.028	0.417	2.402	1.001	0.931	-	acrylic		
10.18	4.63	0.45	0.190	3.32	5.73	7.74	1.038	0.656	1.672	1.097	0.855	0.882	acrylic		
10.18	4.63	0.45	0.340	5.28	6.44	8.40	1.033	0.574	2.114	1.213	1.022	1.070	acrylic		
10.18	4.63	0.45	0.597	8.00	7.46	8.98	1.029	0.517	2.839	1.467	1.306	1.398	acrylic		
10.18	4.63	0.45	0.753	9.22	8.17	8.91	1.029	0.490	3.404	1.668	1.539	1.652	acrylic		
9.98	5.51	0.55	0.158	2.77	5.70	8.01	1.035	0.707	1.656	1.170	0.792	0.827	acrylic		
9.98	5.51	0.55	0.240	3.74	6.43	8.22	1.034	0.638	2.107	1.345	1.014	1.066	acrylic		
9.98	5.51	0.55	0.392	5.82	6.73	8.10	1.035	0.539	2.311	1.244	1.033	1.114	acrylic		
9.98	5.51	0.55	0.403	5.02	8.03	8.39	1.033	0.580	3.288	1.906	1.501	1.590	acrylic		
9.98	5.51	0.55	0.462	6.57	7.04	8.57	1.031	0.533	2.527	1.348	1.108	1.194	acrylic		
9.98	5.51	0.55	0.464	6.27	7.40	8.19	1.034	0.527	2.793	1.473	1.271	-	acrylic		
9.98	5.51	0.55	0.597	7.86	7.60	8.29	1.033	0.490	2.948	1.444	1.279	-	acrylic		
11.06	6.14	0.56	0.183	3.17	5.77	8.11	1.035	0.701	1.697	1.189	0.853	0.869	acrylic		
9.98	5.62	0.56	0.173	2.97	5.83	8.32	1.033	0.698	1.734	1.210	0.797	0.832	acrylic		
9.98	5.62	0.56	0.311	4.82	6.45	8.82	1.030	0.603	2.120	1.278	0.958	1.023	acrylic		
9.98	5.62	0.56	0.465	5.37	8.65	7.48	1.040	0.529	3.813	2.019	1.705	1.803	acrylic		
9.86	6.24	0.63	0.170	2.94	5.80	8.07	1.035	0.690	1.713	1.182	0.795	0.827	acrylic		
9.86	6.24	0.63	0.303	4.77	6.36	8.67	1.031	0.598	2.061	1.232	0.960	1.020	acrylic		
9.86	6.24	0.63	0.596	7.68	7.75	8.95	1.029	0.517	3.065	1.585	1.339	1.448	acrylic		
9.86	6.24	0.63	0.751	8.81	8.53	9.89	1.024	0.527	3.707	1.955	1.628	1.757	acrylic		
12.11	8.10	0.67	0.184	3.17	5.80	8.39	1.033	0.732	1.716	1.256	0.826	0.860	acrylic		
12.11	8.10	0.67	0.222	3.69	6.02	7.87	1.037	0.678	1.850	1.254	0.888	0.922	acrylic		
12.11	8.10	0.67	0.430	6.19	6.94	8.91	1.029	0.594	2.453	1.458	1.113	1.183	acrylic		
12.11	8.10	0.67	0.432	6.42	6.72	8.73	1.030	0.581	2.306	1.340	1.097	1.168	acrylic		
12.11	8.10	0.67	0.592	7.80	7.59	8.84	1.030	0.547	2.939	1.608	1.332	1.425	acrylic		
12.11	8.10	0.67	0.760	9.38	8.10	9.01	1.029	0.520	3.346	1.741	1.506	1.618	acrylic		
12.11	8.10	0.67	0.884	10.20	8.66	9.32	1.027	0.517	3.825	1.978	1.748	1.869	acrylic		
9.83	7.37	0.75	0.181	3.10	5.84	9.02	1.029	0.702	1.740	1.222	0.807	0.833	acrylic		
9.83	7.37	0.75	0.327	4.56	7.18	7.84	1.037	0.577	2.630	1.516	1.191	1.261	acrylic		
9.83	7.37	0.75	0.330	5.05	6.53	8.65	1.031	0.584	2.176	1.271	0.960	1.023	acrylic		

Offset	Gap <i>h</i>	$\beta$	Discharge per unit width						Channel velocity head <i>h<sub>v</sub></i>	Boundary layer velocity head <i>h<sub>v*</sub></i>	Attached uplift $\Delta H$	Total detached uplift $\Delta H + \Delta H_{det}$	Approach flow surface
			Depth <i>y</i>	Velocity <i>V</i>	<i>N</i>	$\alpha$	$\alpha^*$						
			mm	mm	-	m <sup>2</sup> /s	cm	m/s					
9.83	7.37	0.75	0.566	7.55	7.51	8.86	1.030	0.516	2.873	1.483	1.201	1.298	acrylic
9.83	7.37	0.75	0.568	6.71	8.46	9.74	1.025	0.567	3.648	2.069	1.601	1.717	acrylic
9.83	7.37	0.75	0.568	7.08	8.03	9.35	1.027	0.545	3.284	1.790	1.395	1.502	acrylic
9.83	7.37	0.75	0.569	6.08	9.36	8.28	1.033	0.534	4.464	2.384	2.019	2.132	acrylic
9.83	7.37	0.75	0.569	5.86	9.72	8.10	1.035	0.534	4.817	2.574	2.033	2.167	acrylic
4.59	3.56	0.78	0.071	1.58	4.48	8.15	1.034	0.655	1.024	0.671	0.467	0.488	acrylic
4.59	3.56	0.78	0.080	1.68	4.77	7.80	1.037	0.629	1.161	0.731	0.500	0.532	acrylic
4.59	3.56	0.78	0.154	2.74	5.64	8.27	1.034	0.540	1.623	0.877	0.671	0.721	acrylic
4.59	3.56	0.78	0.326	5.00	6.52	8.70	1.031	0.452	2.169	0.980	0.849	0.952	acrylic
9.87	9.59	0.97	0.165	2.86	5.76	8.07	1.035	0.696	1.693	1.179	0.754	0.782	acrylic
9.87	9.59	0.97	0.167	2.88	5.79	8.41	1.032	0.705	1.710	1.205	0.756	0.791	acrylic
9.87	9.59	0.97	0.320	5.03	6.35	8.52	1.032	0.581	2.057	1.196	0.912	0.984	acrylic
9.87	9.59	0.97	0.597	7.96	7.50	9.25	1.027	0.522	2.869	1.497	1.193	1.311	acrylic
9.87	9.59	0.97	0.759	9.17	8.28	9.56	1.026	0.509	3.494	1.780	1.415	1.557	acrylic
11.12	11.34	1.02	0.168	2.91	5.78	8.93	1.029	0.745	1.705	1.271	0.786	0.813	acrylic
11.12	11.34	1.02	0.170	2.94	5.77	8.06	1.035	0.721	1.698	1.225	0.785	0.809	acrylic
11.12	11.34	1.02	0.313	4.87	6.43	8.84	1.030	0.624	2.107	1.315	0.957	1.012	acrylic
11.12	11.34	1.02	0.331	5.06	6.55	8.82	1.030	0.615	2.184	1.343	0.975	-	acrylic
11.12	11.34	1.02	0.452	6.50	6.95	9.17	1.028	0.577	2.466	1.423	1.094	1.167	acrylic
11.12	11.34	1.02	0.599	7.85	7.63	9.13	1.028	0.541	2.968	1.606	1.278	1.367	acrylic
2.23	2.58	1.16	0.107	2.05	5.19	7.95	1.036	0.448	1.375	0.616	0.435	0.500	acrylic
2.23	2.58	1.16	0.251	4.06	6.19	8.30	1.033	0.362	1.956	0.708	0.588	0.710	acrylic
2.23	2.58	1.16	0.589	7.00	8.41	9.55	1.026	0.347	3.609	1.253	0.995	-	acrylic
6.13	7.52	1.23	0.119	2.25	5.31	7.88	1.036	0.632	1.436	0.908	0.588	0.631	acrylic
6.13	7.52	1.23	0.571	6.08	9.40	8.20	1.034	0.447	4.505	2.013	1.624	1.809	acrylic
6.13	7.52	1.23	0.573	7.59	7.54	9.28	1.027	0.456	2.903	1.322	1.035	1.167	acrylic
5.79	7.41	1.28	0.106	1.98	5.33	7.99	1.036	0.653	1.449	0.946	0.603	0.640	acrylic
5.79	7.41	1.28	0.106	1.98	5.33	7.99	1.036	0.653	1.449	0.946	0.579	-	acrylic
5.79	7.41	1.28	0.106	1.98	5.33	7.99	1.036	0.653	1.449	0.946	0.583	-	acrylic
5.79	7.41	1.28	0.106	1.98	5.34	7.79	1.037	0.646	1.452	0.938	0.587	0.630	acrylic
5.79	7.41	1.28	0.140	2.49	5.62	8.21	1.034	0.607	1.609	0.976	0.631	-	acrylic
5.79	7.41	1.28	0.305	4.74	6.43	8.68	1.031	0.498	2.108	1.050	0.793	0.895	acrylic
5.79	7.41	1.28	0.591	7.64	7.74	9.14	1.028	0.441	3.052	1.346	1.094	-	acrylic
10.16	14.69	1.45	0.173	3.01	5.77	8.40	1.033	0.701	1.697	1.189	0.703	0.749	acrylic
10.16	14.69	1.45	0.313	4.85	6.46	8.82	1.030	0.605	2.125	1.286	0.850	0.934	acrylic

Offset	Gap <i>h</i>	$\beta$	Discharge per unit width						Channel velocity head <i>h<sub>v</sub></i>	Boundary layer velocity head <i>h<sub>v*</sub></i>	Attached uplift $\Delta H$	Total detached uplift $\Delta H + \Delta H_{det}$	Approach flow surface
			Depth <i>y</i>	Velocity <i>V</i>	<i>N</i>	$\alpha$	$\alpha^*$						
			mm	mm	-	m <sup>2</sup> /s	cm	m/s					
10.16	14.69	1.45	0.590	7.67	7.69	8.97	1.029	0.523	3.015	1.577	1.153	1.298	acrylic
10.16	14.69	1.45	0.744	8.71	8.54	8.72	1.030	0.492	3.721	1.831	1.380	1.578	acrylic
3.45	5.51	1.60	0.038	1.08	3.52	6.75	1.048	0.632	0.633	0.400	0.216	0.247	acrylic
3.45	5.51	1.60	0.049	1.23	3.98	7.71	1.038	0.633	0.806	0.510	0.258	0.288	acrylic
3.45	5.51	1.60	0.093	1.85	4.99	8.08	1.035	0.555	1.271	0.705	0.384	0.444	acrylic
3.45	5.51	1.60	0.182	3.09	5.90	8.02	1.035	0.456	1.777	0.811	0.499	-	acrylic
3.45	5.51	1.60	0.237	3.89	6.08	8.68	1.031	0.447	1.886	0.842	0.546	-	acrylic
3.45	5.51	1.60	0.248	4.01	6.19	8.45	1.032	0.432	1.952	0.844	0.562	-	acrylic
3.45	5.51	1.60	0.393	5.92	6.64	8.02	1.035	0.358	2.251	0.806	0.616	-	acrylic
3.45	5.51	1.60	0.396	4.79	8.26	7.86	1.037	0.380	3.482	1.323	0.930	1.185	acrylic
3.45	5.51	1.60	0.587	6.61	8.87	7.44	1.040	0.316	4.015	1.271	0.992	-	acrylic
3.45	5.51	1.60	0.600	7.85	7.64	8.10	1.035	0.325	2.978	0.969	0.733	-	acrylic
9.82	17.40	1.77	0.166	2.87	5.77	8.49	1.032	0.706	1.697	1.199	0.655	0.714	acrylic
9.82	17.40	1.77	0.307	4.80	6.39	8.62	1.031	0.594	2.084	1.237	0.793	0.885	acrylic
9.82	17.40	1.77	0.592	7.59	7.79	9.44	1.026	0.536	3.096	1.659	1.094	1.264	acrylic
6.06	11.84	1.95	0.092	1.87	4.93	7.73	1.038	0.671	1.240	0.832	0.444	0.492	acrylic
6.06	11.84	1.95	0.237	3.85	6.16	8.56	1.031	0.540	1.934	1.044	0.655	0.758	acrylic
6.06	11.84	1.95	0.527	6.75	7.81	9.20	1.028	0.468	3.108	1.456	0.992	-	acrylic
6.06	11.84	1.95	0.593	7.23	8.19	9.05	1.029	0.452	3.423	1.548	1.070	1.267	acrylic
3.71	7.76	2.09	0.056	1.35	4.13	7.90	1.036	0.635	0.869	0.552	0.292	0.332	acrylic
3.71	7.76	2.09	0.111	2.11	5.26	7.79	1.037	0.531	1.410	0.749	0.443	0.521	acrylic
3.71	7.76	2.09	0.230	3.78	6.08	8.45	1.032	0.453	1.887	0.855	0.561	0.686	acrylic
3.71	7.76	2.09	0.451	6.34	7.12	9.01	1.029	0.400	2.585	1.034	0.703	-	acrylic
3.71	7.76	2.09	0.598	7.60	7.87	9.05	1.029	0.378	3.158	1.194	0.834	-	acrylic
11.70	31.83	2.72	0.156	2.74	5.68	8.14	1.034	0.756	1.645	1.243	0.549	0.607	acrylic
11.52	31.39	2.73	0.213	3.55	5.99	8.71	1.031	0.699	1.829	1.279	0.618	0.705	acrylic
11.52	31.39	2.73	0.290	4.58	6.33	8.58	1.031	0.637	2.042	1.300	0.688	0.797	acrylic
11.52	31.39	2.73	0.599	7.81	7.68	8.92	1.029	0.541	3.005	1.625	0.948	1.137	acrylic
12.77	37.68	2.95	0.598	7.92	7.55	8.83	1.030	0.554	2.903	1.608	0.826	1.055	acrylic
6.19	19.24	3.11	0.084	1.72	4.90	8.01	1.035	0.706	1.224	0.864	0.343	0.401	acrylic
6.19	19.24	3.11	0.167	2.88	5.82	8.46	1.032	0.599	1.725	1.033	0.482	0.576	acrylic
6.19	19.24	3.11	0.330	5.04	6.55	8.98	1.029	0.511	2.185	1.116	0.573	0.749	acrylic
6.19	19.24	3.11	0.595	7.63	7.79	9.27	1.027	0.456	3.093	1.409	0.763	1.013	acrylic
6.19	19.24	3.11	0.753	8.76	8.60	9.58	1.026	0.447	3.769	1.686	0.920	-	acrylic
6.06	19.47	3.21	0.096	1.93	4.99	7.30	1.042	0.648	1.267	0.821	0.359	0.424	acrylic

Offset	Gap <i>h</i>	<i>s</i>	$\beta$	Discharge per unit width						Channel velocity head <i>h<sub>v</sub></i>	Boundary layer velocity head <i>h<sub>v*</sub></i>	Attached uplift $\Delta H$	Total detached uplift $\Delta H + \Delta H_{det}$	Approach flow surface						
				Depth			Velocity													
				<i>q</i>	<i>y</i>	<i>V</i>	<i>N</i>	$\alpha$	$\alpha^*$											
mm	mm	-		m <sup>2</sup> /s	cm	m/s	-	-	-	m	m	m	m	m	-					
6.06	19.47	3.21	0.572	7.66	7.46	8.74	1.030	0.431	2.841	1.225	0.707	0.928	0.928	acrylic						
2.61	9.21	3.53	0.038	1.08	3.55	7.16	1.043	0.576	0.644	0.371	0.160	0.195	0.195	acrylic						
2.61	9.21	3.53	0.148	2.60	5.71	8.30	1.033	0.450	1.664	0.749	0.366	0.480	0.480	acrylic						
2.61	9.21	3.53	0.590	7.56	7.81	9.04	1.029	0.337	3.107	1.046	0.624	-	-	acrylic						
12.32	50.70	4.11	0.213	3.60	5.93	8.54	1.032	0.708	1.791	1.268	0.451	0.535	0.535	acrylic						
12.32	50.70	4.11	0.348	5.24	6.63	8.93	1.029	0.633	2.241	1.418	0.577	0.713	0.713	acrylic						
12.32	50.70	4.11	0.590	7.62	7.74	8.89	1.029	0.557	3.057	1.702	0.734	0.987	0.987	acrylic						
12.31	50.90	4.14	0.159	2.80	5.67	8.32	1.033	0.768	1.639	1.258	0.387	0.452	0.452	acrylic						
12.21	56.12	4.60	0.165	2.89	5.71	8.08	1.035	0.752	1.662	1.249	0.351	0.418	0.418	acrylic						
12.11	56.26	4.64	0.223	3.76	5.95	8.53	1.032	0.693	1.804	1.251	0.414	0.493	0.493	acrylic						
12.11	56.26	4.64	0.355	5.32	6.67	8.72	1.030	0.620	2.268	1.405	0.521	0.640	0.640	acrylic						
12.11	56.26	4.64	0.589	7.62	7.73	9.11	1.028	0.561	3.050	1.711	0.664	0.867	0.867	acrylic						
11.90	62.68	5.27	0.134	2.40	5.57	8.14	1.034	0.799	1.583	1.265	0.266	0.316	0.316	acrylic						
11.90	62.68	5.27	0.280	4.44	6.32	8.66	1.031	0.653	2.037	1.331	0.412	0.516	0.516	acrylic						
11.90	62.68	5.27	0.592	7.63	7.75	9.06	1.028	0.556	3.067	1.704	0.603	0.839	0.839	acrylic						
11.66	62.90	5.39	0.124	2.30	5.38	8.39	1.033	0.810	1.477	1.196	0.239	0.277	0.277	acrylic						
11.66	62.90	5.39	0.214	3.59	5.98	8.95	1.029	0.706	1.821	1.286	0.339	0.420	0.420	acrylic						
11.66	62.90	5.39	0.394	5.77	6.83	8.79	1.030	0.597	2.379	1.420	0.444	0.605	0.605	acrylic						
11.66	62.90	5.39	0.599	7.82	7.67	8.61	1.031	0.531	2.996	1.592	0.559	0.782	0.782	acrylic						
11.66	62.90	5.39	0.746	8.74	8.54	9.13	1.028	0.530	3.716	1.971	0.718	0.994	0.994	acrylic						
3.65	19.93	5.46	0.054	1.32	4.11	7.37	1.041	0.616	0.860	0.530	0.149	0.190	0.190	acrylic						
3.65	19.93	5.46	0.084	1.77	4.78	7.71	1.038	0.562	1.166	0.655	0.204	0.271	0.271	acrylic						
3.65	19.93	5.46	0.175	3.01	5.80	8.42	1.032	0.487	1.715	0.835	0.299	-	-	acrylic						
3.65	19.93	5.46	0.314	4.86	6.46	8.84	1.030	0.428	2.129	0.911	0.348	-	-	acrylic						
3.65	19.93	5.46	0.582	6.62	8.78	9.28	1.027	0.402	3.935	1.583	0.630	-	-	acrylic						
3.65	19.93	5.46	0.600	7.84	7.66	9.35	1.027	0.384	2.991	1.149	0.460	-	-	acrylic						
6.38	37.59	5.90	0.106	2.03	5.23	7.85	1.037	0.666	1.396	0.930	0.211	0.266	0.266	acrylic						
6.38	37.59	5.90	0.370	4.84	7.65	8.44	1.032	0.502	2.980	1.497	0.486	-	-	acrylic						
6.38	37.59	5.90	0.390	5.73	6.80	8.66	1.031	0.482	2.356	1.135	0.378	-	-	acrylic						
6.38	37.59	5.90	0.576	6.46	8.91	8.64	1.031	0.461	4.052	1.869	0.644	0.959	0.959	acrylic						
6.38	37.59	5.90	0.580	7.63	7.60	8.58	1.031	0.433	2.949	1.276	0.456	0.690	0.690	acrylic						
6.23	37.69	6.05	0.119	2.24	5.32	8.63	1.031	0.661	1.446	0.955	0.223	0.288	0.288	acrylic						
6.23	37.69	6.05	0.235	3.69	6.37	9.07	1.028	0.571	2.071	1.182	0.340	0.459	0.459	acrylic						
6.23	37.69	6.05	0.237	3.89	6.08	9.03	1.029	0.560	1.886	1.055	0.312	0.436	0.436	acrylic						
6.23	37.69	6.05	0.566	5.91	9.58	8.72	1.030	0.475	4.680	2.225	0.702	-	-	acrylic						

Offset	Gap <i>h</i>	<i>s</i>	$\beta$	Discharge per unit width						Channel velocity head <i>h<sub>v</sub></i>	Boundary layer velocity head <i>h<sub>v*</sub></i>	Attached uplift $\Delta H$	Total detached uplift $\Delta H + \Delta H_{det}$	Approach flow surface
				Depth <i>y</i>	Velocity <i>V</i>	<i>N</i>	$\alpha$	$\alpha^*$						
				m	mm	m <sup>2</sup> /s	cm	m/s	-	-	-	-	-	-
6.23	37.69	6.05	0.570	7.59	7.51	9.14	1.028	0.453	2.874	1.301	0.442	-	-	acrylic
3.00	20.32	6.78	0.037	1.06	3.49	6.83	1.047	0.600	0.620	0.372	0.094	0.108	-	acrylic
3.00	20.32	6.78	0.048	1.24	3.86	7.32	1.041	0.582	0.760	0.442	0.116	0.143	-	acrylic
3.00	20.32	6.78	0.125	2.23	5.60	7.88	1.036	0.483	1.597	0.771	0.232	0.302	-	acrylic
3.00	20.32	6.78	0.319	4.94	6.46	8.98	1.029	0.403	2.129	0.859	0.306	-	-	acrylic
3.00	20.32	6.78	0.595	7.66	7.77	8.77	1.030	0.340	3.078	1.047	0.409	-	-	acrylic
11.06	75.57	6.83	0.148	2.61	5.66	8.08	1.035	0.752	1.632	1.228	0.229	0.270	-	acrylic
11.06	75.57	6.83	0.312	4.85	6.43	8.81	1.030	0.623	2.105	1.311	0.351	0.454	-	acrylic
11.06	75.57	6.83	0.589	7.59	7.76	9.36	1.027	0.554	3.072	1.701	0.515	0.709	-	acrylic
11.06	75.57	6.83	0.755	8.82	8.57	9.21	1.028	0.523	3.742	1.956	0.617	0.872	-	acrylic
3.71	27.12	7.30	0.040	1.11	3.60	6.99	1.045	0.654	0.662	0.433	0.083	0.098	-	acrylic
3.71	27.12	7.30	0.050	1.27	3.96	7.27	1.042	0.627	0.798	0.501	0.103	0.127	-	acrylic
3.71	27.12	7.30	0.066	1.52	4.31	7.11	1.044	0.575	0.947	0.545	0.132	0.169	-	acrylic
3.71	27.12	7.30	0.091	1.84	4.93	7.45	1.040	0.546	1.241	0.678	0.166	0.223	-	acrylic
3.71	27.12	7.30	0.112	2.07	5.39	7.98	1.036	0.543	1.482	0.804	0.195	0.267	-	acrylic
3.71	27.12	7.30	0.151	2.67	5.65	8.55	1.032	0.516	1.629	0.841	0.229	0.306	-	acrylic
3.71	27.12	7.30	0.207	3.48	5.94	8.67	1.031	0.475	1.800	0.855	0.254	0.372	-	acrylic
3.71	27.12	7.30	0.211	3.52	5.99	8.34	1.033	0.460	1.827	0.840	0.244	0.384	-	acrylic
3.71	27.12	7.30	0.256	4.13	6.19	8.62	1.031	0.446	1.952	0.870	0.276	-	-	acrylic
3.71	27.12	7.30	0.313	4.83	6.47	8.63	1.031	0.423	2.135	0.902	0.293	-	-	acrylic
3.71	27.12	7.30	0.358	5.34	6.70	8.72	1.030	0.412	2.289	0.942	0.309	-	-	acrylic
3.71	27.12	7.30	0.387	5.71	6.78	8.74	1.030	0.403	2.345	0.946	0.316	-	-	acrylic
3.71	27.12	7.30	0.478	6.66	7.17	9.05	1.029	0.395	2.623	1.036	0.347	-	-	acrylic
3.71	27.12	7.30	0.591	7.67	7.70	8.76	1.030	0.365	3.026	1.105	0.384	-	-	acrylic
3.71	27.12	7.30	0.742	8.69	8.54	8.90	1.029	0.356	3.719	1.323	0.458	-	-	acrylic
6.57	50.74	7.73	0.089	1.77	5.03	6.94	1.046	0.682	1.290	0.879	0.154	0.183	-	acrylic
6.57	50.74	7.73	0.098	1.88	5.21	7.84	1.037	0.693	1.387	0.961	0.166	0.198	-	acrylic
6.57	50.74	7.73	0.164	2.86	5.73	8.21	1.034	0.604	1.677	1.013	0.234	0.295	-	acrylic
6.57	50.74	7.73	0.333	5.11	6.51	8.58	1.031	0.503	2.160	1.087	0.320	0.453	-	acrylic
6.57	50.74	7.73	0.589	7.60	7.75	8.80	1.030	0.447	3.064	1.370	0.440	-	-	acrylic
3.37	27.58	8.17	0.036	1.03	3.45	7.76	1.037	0.673	0.608	0.409	0.069	0.084	-	acrylic
3.37	27.58	8.17	0.071	1.56	4.58	7.65	1.038	0.570	1.068	0.609	0.127	0.166	-	acrylic
3.37	27.58	8.17	0.142	2.57	5.53	7.99	1.036	0.483	1.558	0.753	0.197	0.278	-	acrylic
3.37	27.58	8.17	0.303	4.72	6.41	8.58	1.031	0.410	2.097	0.860	0.263	-	-	acrylic
3.37	27.58	8.17	0.588	7.60	7.74	9.09	1.028	0.368	3.054	1.123	0.363	-	-	acrylic

Offset	Gap <i>h</i>	$\beta$	Discharge per unit width						Channel velocity head <i>h<sub>v</sub></i>	Boundary layer velocity head <i>h<sub>v*</sub></i>	Attached uplift $\Delta H$	Total detached uplift $\Delta H + \Delta H_{det}$	Approach flow surface	
			Depth	Velocity <i>y</i>	<i>V</i>	<i>N</i>	$\alpha$	$\alpha^*$						
			mm	mm	-	m <sup>2</sup> /s	cm	m/s	-	-	-	m	m	-
3.37	27.58	8.17	0.732	8.63	8.47	9.10	1.028	0.353	3.661	1.293	0.445	-	0.178	acrylic
8.40	75.83	9.03	0.119	2.18	5.44	8.00	1.035	0.724	1.510	1.094	0.155	-	0.288	acrylic
8.40	75.83	9.03	0.236	3.93	6.02	8.88	1.030	0.611	1.847	1.129	0.224	-	0.403	acrylic
8.40	75.83	9.03	0.373	5.54	6.72	8.71	1.031	0.538	2.303	1.239	0.281	-	0.083	acrylic
8.40	75.83	9.03	0.590	7.62	7.74	9.17	1.028	0.500	3.055	1.526	0.391	-	-	acrylic
3.84	37.34	9.71	0.045	1.18	3.78	6.81	1.047	0.639	0.727	0.464	0.068	-	-	acrylic
3.84	37.34	9.71	0.165	2.89	5.72	8.26	1.034	0.497	1.669	0.829	0.191	-	-	acrylic
3.84	37.34	9.71	0.588	7.63	7.71	9.06	1.028	0.382	3.033	1.160	0.328	-	-	acrylic
3.45	43.98	12.75	0.056	1.35	4.15	7.59	1.039	0.606	0.877	0.532	0.064	-	-	acrylic
3.45	43.98	12.75	0.115	2.19	5.27	7.88	1.036	0.513	1.415	0.725	0.118	-	-	acrylic
3.45	43.98	12.75	0.230	3.77	6.10	8.77	1.030	0.454	1.898	0.862	0.175	-	-	acrylic
3.45	43.98	12.75	0.465	6.47	7.19	8.91	1.029	0.384	2.636	1.012	0.230	-	-	acrylic
3.45	43.98	12.75	0.598	7.62	7.85	8.74	1.030	0.356	3.142	1.119	0.268	-	-	acrylic
3.26	55.89	17.15	0.150	2.64	5.70	8.35	1.033	0.487	1.654	0.806	0.109	-	-	acrylic
3.26	55.89	17.15	0.251	4.07	6.16	8.58	1.031	0.427	1.937	0.827	0.131	-	-	acrylic
3.26	55.89	17.15	0.566	5.89	9.60	8.81	1.030	0.384	4.697	1.804	0.321	-	-	acrylic
3.26	55.89	17.15	0.571	7.60	7.52	9.07	1.028	0.363	2.882	1.046	0.208	-	-	acrylic
3.06	56.30	18.38	0.056	1.36	4.12	7.86	1.037	0.588	0.865	0.508	0.049	-	-	acrylic
3.06	56.30	18.38	0.115	2.17	5.28	8.20	1.034	0.505	1.424	0.719	0.086	-	-	acrylic
3.06	56.30	18.38	0.224	3.71	6.05	8.37	1.033	0.423	1.866	0.789	0.127	-	-	acrylic
3.06	56.30	18.38	0.372	5.47	6.80	8.77	1.030	0.384	2.360	0.907	0.161	-	-	acrylic
3.06	56.30	18.38	0.598	7.46	8.02	7.89	1.036	0.308	3.282	1.011	0.212	-	-	acrylic
1.37	44.08	32.08	0.037	1.06	3.51	7.58	1.039	0.463	0.627	0.290	0.028	-	-	acrylic
1.37	44.08	32.08	0.070	1.55	4.51	7.80	1.037	0.408	1.035	0.422	0.050	-	-	acrylic
1.37	44.08	32.08	0.140	2.53	5.54	7.93	1.036	0.344	1.565	0.539	0.084	-	-	acrylic
1.37	44.08	32.08	0.279	4.34	6.42	8.62	1.031	0.310	2.104	0.653	0.123	-	-	acrylic
1.37	44.08	32.08	0.598	7.62	7.85	8.88	1.030	0.265	3.145	0.834	0.188	-	-	acrylic
10.40	0.76	0.073	0.138	2.56	5.38	8.22	1.034	0.744	1.473	1.096	-	0.854	acrylic	
10.18	4.63	0.45	0.125	2.34	5.35	7.43	1.040	0.743	1.461	1.086	-	0.746	acrylic	
9.56	4.55	0.48	0.016	0.67	2.45	7.52	1.040	1.199	0.305	0.366	-	0.169	acrylic	
9.56	4.55	0.48	0.026	0.84	3.05	7.17	1.043	1.102	0.474	0.522	-	0.248	acrylic	
9.56	4.55	0.48	0.041	1.10	3.68	6.64	1.049	0.985	0.691	0.681	-	0.365	acrylic	
9.56	4.55	0.48	0.083	1.71	4.83	7.45	1.040	0.823	1.190	0.979	-	0.593	acrylic	
9.87	9.59	0.97	0.091	1.82	4.98	7.86	1.037	0.820	1.263	1.037	-	0.610	acrylic	
8.40	75.83	9.03	0.091	1.82	4.98	7.75	1.038	0.769	1.266	0.974	-	0.145	acrylic	

Offset <i>h</i>	Gap <i>s</i>	$\beta$	Discharge per unit width			Depth <i>y</i>	Velocity <i>V</i>	<i>N</i>	$\alpha$	$\alpha^*$	Channel velocity head <i>h<sub>v</sub></i>	Boundary layer velocity head <i>h<sub>v*</sub></i>	Attached uplift $\Delta H$	Total detached uplift $\Delta H + \Delta H_{det}$	Approach flow surface	
			<i>q</i>	<i>m</i> <sup>2</sup> /s	<i>cm</i>											
mm	mm	-														
20.73	7.23	0.35	0.393	6.26	6.29	5.00	1.080	0.557	2.015	1.122	0.920	0.969	0.969	ROUGH		
20.73	7.23	0.35	0.746	9.34	7.99	5.01	1.080	0.438	3.259	1.428	1.326	1.428	1.428	ROUGH		
2.98	6.37	2.14	0.598	8.08	7.40	5.78	1.063	0.191	2.791	0.534	0.408	-	-	ROUGH		
2.36	10.91	4.61	0.237	4.17	5.68	4.98	1.080	0.192	1.644	0.316	0.168	-	-	ROUGH		
2.36	10.91	4.61	0.390	5.99	6.51	5.09	1.078	0.160	2.160	0.346	0.186	-	-	ROUGH		
4.73	37.82	8.00	0.137	3.53	3.90	5.61	1.066	0.364	0.774	0.281	0.081	-	-	ROUGH		

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